[3]Kodiyalam, S., and Vanderplaats, G. N., "Shape Optimization of Three Dimensional Continuum Structures via Force Approximation Techniques," *AIAA Journal*, Vol. 27, No. 9, Sept. 1989, pp. 1256–1263.

[4]"DOT Users Manual," VMA Engineering, Santa Barbara, CA, 1986.

Optimal Design for Plate Structures with Buckling Constraints

Chien-Chang Lin and Ine-Wei Liu†
National Chung-Hsing University, Taichung, Taiwan

Introduction

THE first use of the optimality criterion as a redesign equation was due to Venkayya et al.,[1] and represented a significant development in structural optimization. However, the assumption of linear size-stiffness relations in their algorithm was a major restriction. Discretized optimality criteria methods that treat system buckling and minimum size constraints have been reported,[2,3] but these studies fail to treat system buckling in parallel with other common behavioral constraints such as static stress and displacement limitations. Furthermore, they completely ignore the more important problems associated with preventing local buckling failure modes. On the other hand, Giles[4] includes consideration of buckling failure modes in the design of wing structures. This Note presents a general redesign approach that includes the usual stress, displacement, member size, and buckling constraints as discussing in Ref. 5. The optimality criterion derived for all the constraints imposed on the structure is equivalent to the Kuhn-Tucker conditions of nonlinear mathematical programming. However, the criterion approach is formulated in a general form with all the constraints being combined together or in a simple form with single constraint. An optimization computer code ARS 4 (automatic resizing system 4) has been developed and applied to some practical examples.

Analysis

The objective of our problem is to find the size vector t of ne elements t_i $(i=1,2,\ldots,ne)$, which will minimize the plate structures weight function $W(t)$ subject to

Behavior constraints:

$$g_j(t)=G_j(t)-\hat{G}_j\leq 0,\quad j=1,\ldots,nc \tag{1a}$$

Size constraints:

$$t_i^L\leq t_i\leq t_i^U,\quad i=1,\ldots,ne \tag{1b}$$

The behavior quantities $G_j(t)$ are displacements, stresses, and critical buckling load parameter, whereas $\hat{G}_j$ are limits. It is seldom desirable to have each t_i as an independent design variable. Design variable linking can be used to reduce the number of design variables to nv, $nv\leq ne$; therefore, the actual design variables, denoted by $x=\{x_1, x_2,\ldots, x_{nv}\}^T$, are related to t_i by a relationship as

$$t_i=\zeta_{im}x_m \qquad (i=1,\ldots,ne;\qquad m=1,\ldots,nv) \tag{2}$$

It is well known to define the linear instability of a structure by the eigenvalue problem, i.e.,

$$(K-\mu_j K_G)\tilde{\eta}_j=0 \tag{3}$$

where K is the structural stiffness matrix, K_G the geometric stiffness matrix, and $\tilde{\eta}_j$ the eigenvector associated with the jth eigenvalue μ_j. The eigenvalue μ_j can be obtained by the Rayleigh quotient as

$$\mu_j=\frac{\tilde{\eta}_j^T K\tilde{\eta}_j}{\tilde{\eta}_j^T K_G\tilde{\eta}_j},\qquad j=1,\ldots,nd \tag{4}$$

To solve the optimization problem, we first select the initial point in the feasible region and then, after new iterations, move the design point toward the constrained boundary. Assume there are na constraints encountered during the optimization process, i.e.,

$$g_j(x)=0,\quad j=1,\ldots,na \qquad \text{(active constraints)} \tag{5}$$

$$g_j(x)\leq 0,\quad j=na+1,\ldots,nc \qquad \text{(passive constraints)} \tag{6}$$

By the Kuhn-Tucker optimality criterion, at the optimum point x^*, we may find a set of nonnegative Lagrange multipliers $\tilde{\lambda}=\{\lambda_1,\lambda_2,\ldots,\lambda_{na}\}^T$, such that

$$-\frac{\partial W(x^*)}{\partial x_i}=\sum_{j=1}^{na}\lambda_j\frac{\partial g_j(x^*)}{\partial x_i},\quad i=1,\ldots,nv \tag{7}$$

The gradient projection method can be used to solve Eq. (7) instead of the approximate approach. If the optimum point has been found, then Eq. (7) must be achieved. In general, during the designing iterations, the ratio of the right side to the left side of each part in Eq. (7) might not be 1 but will approach 1. We define multiplier update factors r_i in the process of optimization by

$$r_i=-\sum_{j=1}^{na}\lambda_j\frac{\partial_{gi}/\partial x_i}{\partial W/\partial x_i},\qquad i=1,\ldots,nv \tag{8}$$

and use them as modification factors for the iterations such that

$$x_i^{k+1}=x_i^k r_i,\qquad i=1,\ldots,nv \tag{9}$$

assuming that all the design variables are active and k is the iteration number. As the values of r_i converge to one, the optimum point is reached.

Design sensitivity analysis plays a central role in structual optimization, since virtually all optimization methods require the computation of derivatives of structural response quantities with respect to design variables. In optimality criterion methods, these derivatives are required in calculating the Lagrange multipliers. The detailed sensitivity analysis for the displacements, stresses, and buckling constraints used in this study can be found in Ref. 5.

Design Algorithm

The most important design requirement in a structure is to satisfy the appropriate strength criterion in each element. In practice, the strength criterion is satisfied by using the fully stressed design (FSD) concept, which is one of the early optimality criteria. When the design point is far from the optimal point, the use of FSD can made the design variables rapidly approach the vicinity of the optimal point. As the designing

Received April 5, 1988; revision received Sept. 23, 1988.

*Professor, Institute of Applied Mathematics. Member AIAA.
†Graduate Student, Institute of Applied Mathematics.

point nears the optimal point, then applying the optimality criterion (OC) to optimize the design variables is the most economical algorithm. The constraint of FSD algorithm corresponding to general plate elements is obtained from the modified Von Mises yield criterion as

$$\left(\frac{\sigma_x}{\sigma^*}\right)^2+\left(\frac{\sigma_y}{\sigma^*}\right)^2-\frac{\sigma_x\sigma_y}{(\sigma^*)^2}+\left(\frac{\tau_{xy}}{\tau^*}\right)^2=1 \tag{10}$$

where σ^* and τ^* are the allowable normal and shear stresses, respectively.

When the local instability criterion is used, one may recheck the design at each component level. This buckling criterion was modified to take account of biaxial in-plane loading acting on the panel in conjunction with in-plane shear loading. The local buckling constraints are based on the simply supported boundary conditions along the lines where adjacent plate elements intersect. The general form of the compressive buckling stress is

$$\sigma_{\rm cr}=k\frac{\pi^2E}{12(1-\nu^2)}\left(\frac{t}{w_e}\right)^2 \tag{11}$$

where w_e is the transverse (shorter) width of the plate, t the thickness, and k the buckling coefficient. If all loads are applied simultaneously, the buckling criterion is taken as

$$\frac{\sigma_x}{(\sigma_x)_{\rm cr}}+\frac{\sigma_y}{(\sigma_y)_{\rm cr}}+\frac{\tau_{xy}^2}{(\tau_{xy})_{\rm cr}^2}=1 \tag{12}$$

To prevent intermediate designs departing excessively from the critical state and in turn to have a stabilizing influence on the convergence of the design process, it is frequently advantageous to incorporate the so-called uniform scaling operation in the design procedure, where all the design variables are scaled by a factor Ω, i.e.,

$$x_m'=\Omega x_m, \qquad m=1,\ldots,nv \tag{13}$$

The scale factor is calculated from the condition that the scaled design x should be critical, i.e.,

$$\Omega=\mathrm{Max}\{S_{\max},R_{\max},T_{\max},L\} \tag{14}$$

where $S_{\max}$, $R_{\max}$, $T_{\max}$, and L are most critical stess ratio, displacement ratio, size ratio, and buckling load ratio for all elements in a structures, respectively; each ratio is the value corresponding to the current design data divided by the allowable value.

The optimization computer code ARS 4 developed in this study is composed of a finite-element module, optimal design module, and fully stressed design module. Each module can be used alone or in association with others, depending on the user, so the use of ARS 4 is very convenient and flexible. ARS 4 is written in ANSI FORTRAN-77 and is developed on the VAX-11/780) computer system, which can be adopted easily by other computer systems possessing the FORTRAN-77 compiler. The major steps in ARS 4 are as follows:

1) Input finite-element mesh data.
2) Perform structural weight sensitivity analysis.
3) Analyze the structure with finite-element method.
4) Perform fully stressed design once or twice.
5) Move the design point to the constraint boundary by uniform scaling.
6) Determine the active constraints and perform the sensitivity analysis.
7) Update Lagrange multipliers and design variables by the Kuhn-Tucker condition.
8) If convergent, go the step 9; otherwise return to step 5.
9) Take uniform scaling.
10) Reanalyze the structure and print out results.

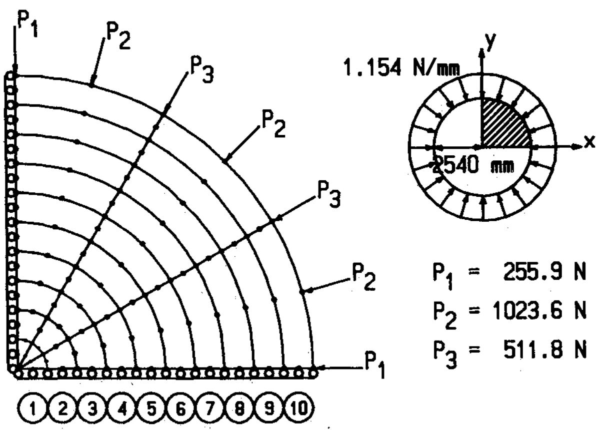


Fig. 1 Simple circular plate under uniform in-plane compression.

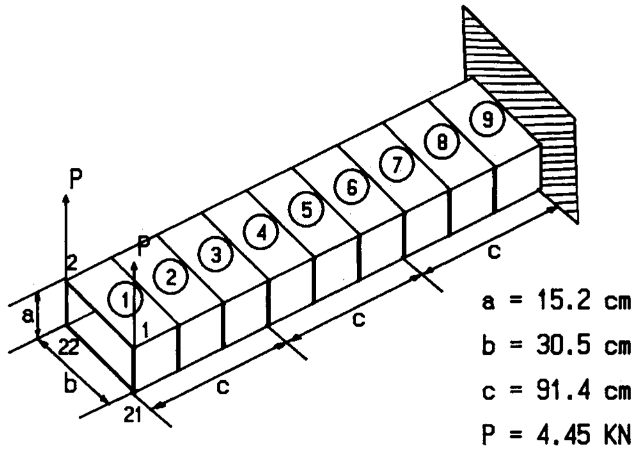
Examples and Discussions

Example 1: Simple Circular Plate

For a simple circular plate under uniform compression, it is sufficient to model only a quarter of the plate with an eight-node isoparametric plate element as shown in Fig. 1. The material properties are $E=6895$ MPa (10^3 ksi), $\nu=0.3$, and $\rho=27.7$ g/cm^3. The allowable stress is 344.75 MPa (50 ksi) and the deisgn variables are linking to 10 for axisymmetry. After using the FSD once and OC twice by ARS 4, the minimum weight converges to 1756 kg. The minimum weight obtained by Kinusalass and Reddy[6] using DESAP 2 with five iterations is 1727 kg, which is lower than that we have. This is

Table 1 Design history of element thickness (mm) for example 1

Design var. no.	Initial data	ARS 4 (present)			DESAP 2 (Ref. 6)				
		FSD	OC		OC				
	0	1	1	2	1	2	3	4	5
1	14.11	1.63	19.26	19.26	16.87	18.06	18.33	18.35	18.34
2	14.11	1.60	18.96	18.94	16.61	17.77	18.06	18.09	18.08
3	14.11	1.55	18.47	18.44	16.12	17.27	17.63	17.69	17.70
4	14.11	1.47	17.67	17.67	15.44	16.54	16.96	17.07	17.10
5	14.11	1.37	16.89	16.88	14.64	15.59	16.06	16.22	16.27
6	14.11	1.25	15.75	15.70	13.81	14.40	14.90	15.11	15.19
7	14.11	1.11	14.01	14.00	13.01	13.01	13.43	13.69	13.81
8	14.11	0.97	12.10	12.05	12.31	11.52	11.60	11.86	12.01
9	14.11	0.83	9.44	9.45	11.76	10.15	9.42	9.41	9.58
10	14.11	0.72	5.66	5.59	11.39	9.21	7.52	6.43	6.02
Weight, ton	1.978	0.151	1.758	1.756	1.836	1.769	1.738	1.726	1.727



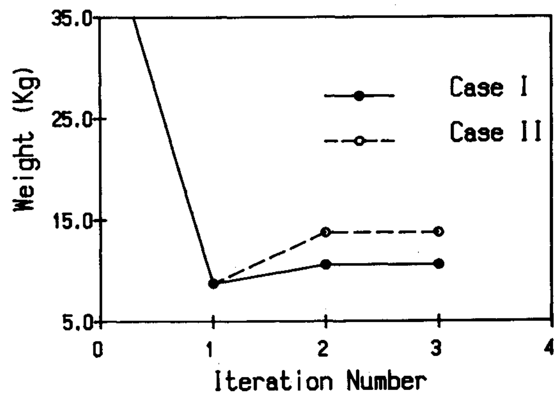


Fig. 2 Cantilever box beam.

because DESAP 2 used the LCCT-9 element, which is stiffer than eight-node isoparametric element and the fact that, in Ref. 6, this plate was modeled for only a 2 deg angle sector. Reanalyzing the plate with thickness obtained in Ref. 6 by ARS 4, it is found that the buckling load factor is only 0.832, i.e., buckling has occurred under the design of Ref. 6. A comparison of the design history of element thickness is shown in Table 1.

Example 2: Cantilever Box Beam

A cantilever box beam is constructed by 18 quadrilateral membrane elements for the top and bottom skins, 18 shear panels for the vertical sides, and 18 bars under two transverse concentrated loads at nodes 1 and 2, as shown in Fig. 2. The material properties are $E = 72.4$ GPa, $\rho = 2.77$ g/cm^3, and $\nu = 0.3$ for all the elements. The allowable stress for bars and membranes is 172.4 MPa and for shear panels is 55.2 MPa. The minimum size for membranes and shear panels is 0.25 mm and for bars is 64.5 mm^2. Two cases of constraint conditions are investigated.

Case I—Stress and minimum size constraints are imposed on all of the elements; deflection constraint at the tip is 10 cm. Applying FSD once and OC twice by ARS 4, the minimum weight converges to 10.66 kg. The deflections at nodes 1 and 2 are just equal to 10 cm.

Table 2 Optimum design values of cantilever box beam

Design	Block no.	Case I	Case II
Membranes,	1	0.257	0.791
mm	2	0.615	1.307
	3	1.027	1.515
	4	1.438	1.831
	5	1.849	2.354
	6	2.260	2.879
	7	2.665	3.393
	8	3.115	3.975
	9	3.279	4.182
Panels, mm	1-9	0.642	0.818
Bars, mm^2	1-9	64.50	64.50
Weight, kg		10.66	13.75

Case II—Buckling constraints are added to case I. Using FSD once and OC twice by ARS 4, the minimum weight converges to 13.75 kg. The deflections at nodes 1 and 2 are now only 8.05 cm, i.e., the increased thickness for the beam satisfies the modified in-plane forces of the buckling formula.

The minimum weight histories and the optimum values for cases I and II are given in Fig. 2 and Table 2.

Conclusions

The conclusions can be summarized as follows.

1) The algorithm of combining OC technique with FSD method reduces the redesign iteration numbers and CPU times. This algorithm has improved the weakness of the OC technique in which the initial design variables need to be chosen near the optimum point.

2) The Kuhn-Tucker optimality condition is applied directly in the optimization process. No line search is needed and thus no structural behavior approximation is required.

3) Practical applications show that the active constraints are always a small portion of entire constraints. Thus, a large-scale optimal design problem may be solved easily.

4) The algorithm ARS 4 presented in this Note can be applied to other element types and other behavioral constraints as soon as the sensitivity analysis and stress ratioing have been done.

References

[1]Venkayya, V. B., Knot, N. S., Tischler, V. A., and Taylor, R. F., "Design of Optimum Structures for Dynamic Loads," *Proceedings of Third Conference and Matrix Methods and Structural Mechanics,* Wright-Patterson AFB, OH, 1971, pp. 619-658.

[2]Kiusalass, J., "Optimum Design of Structures with Buckling Constraints," *International Journal of Solids and Structures,* Vol. 9, 1973, pp. 863-878.

[3]Khot, N. S., Venkayya, V. B., and Berke, L, "Optimum Structural Design with Stability Constraints," *International Journal for Numerical Methods in Engineering,* Vol. 10, 1976, pp. 1097-1114.

[4]Giles, G. L., "Procedures for Automating Aircraft Wing Structural Design," *Journal fo the Structural Division, ASCE,* Vol. 97, 1971, pp. 99-113.

[5]Lin, C. C., and Liu, I. W., "Optimal Design Based on Optimality Criterion for Frame Structures Including Buckling Constraints," *Computers and Structures* (to be published).

[6]Kiusalaas, J., and Reddy, G. B., *"DESAP 2-A Structural Design Program with Stress and Buckling Constraints,"* NASA CR-2797 and 2798, March 1977.